MAT 155 –PreCalculus Mathematics Common Formulas

Factoring Cubic Polynomials

Sum of Two Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Difference of Two Cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Distance and Midpoint Formulas

Distance, d, between two endpoints (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of the line segment with Endpoints (x_1, y_1) and (x_2, y_2) is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Quadratic Functions

The quadratic function, $f(x) = ax^2 + bx + c$ has vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

The quadratic function, $f(x) = a(x-h)^2 + k$ has vertex (h, k)

The Remainder Theorem

If the polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

The Factor Theorem

Let $f(x)$ be a polynomial. If $f(c) = 0$ then $x - c$ is a factor of $f(x)$.

If $x - c$ is a factor of $f(x)$ then $f(c) = 0$.

Asymptotes of Rational Functions

Finding the Horizontal Asymptotes

Consider the rational function,

$$f(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_2 x^2 + b_1 x + b_0}, \quad a_n \neq 0, \quad b_m \neq 0$$

1. If $n < m$, the line $y = 0$ is the horizontal asymptote of the graph of f.
2. If $n = m$, the line $y = \frac{a_n}{b_m}$ is the horizontal asymptote of the graph of f.
3. If $n > m$, the graph of f has no horizontal asymptote.

Finding Oblique (Slant) Asymptotes

Let $N(x)$ and $D(x)$ be polynomials and consider the rational function $f(x) = \frac{N(x)}{Q(x)}$.

If the degree of $N(x)$ is *one more than* the degree of $D(x)$, then graph of $f(x)$ has an oblique (slant) asymptote. To find the equation for the oblique (slant) asymptote first perform long division to obtain

$$f(x) = \frac{N(x)}{D(x)} = mx + b + \frac{remainder}{D(x)}$$

The oblique (slant) asymptote has equation $y = mx + b$.

Compound Interest

After t years, the balance, A, in an account with principal P and annual interest rate, r, is given by

1. $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$, for n compounding periods per year.
2. $A(t) = Pe^{rt}$, for continuously compounding interest.

Change–of-Base Formula for Logarithms

For any positive real numbers M, b, and n, where $n \neq 1$, and $b \neq 1$,

$$\log_b M = \frac{\log_n M}{\log_n b}$$

Trigonometric Identities

Pythagorean Identities

$\tan^2(t) + 1 = \sec^2(t)$ $1 + \cot^2(t) = \csc^2(t)$

Sum and Difference formulas for Sine

$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

Sum and Difference formulas for Cosine

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

Sum and Difference formulas for Tangent

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Double Angle Formulas

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

Power-Reducing Formulas

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2(\theta) = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} = \frac{1-\cos(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{1+\cos(\theta)}$$

Product to Sum Formulas

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\cos(\alpha)\sin(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\cos(\alpha)+\cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha)-\cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha)+\sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha)-\sin(\beta) = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

The Unit Circle