Factoring Cubic Polynomials

**Sum of Two Cubes**
\[a^3 + b^3 = (a + b)(a^2 - ab + b^2)\]

**Difference of Two Cubes**
\[a^3 - b^3 = (a - b)(a^2 + ab + b^2)\]

Distance and Midpoint Formulas

**Distance**, \(d\), between two endpoints \((x_1, y_1)\) and \((x_2, y_2)\) is given by
\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

**Midpoint** of the line segment with Endpoints \((x_1, y_1)\) and \((x_2, y_2)\) is given by
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

Quadratic Functions

The quadratic function, \(f(x) = ax^2 + bx + c\) has vertex \(\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)\)

The quadratic function, \(f(x) = a(x - h)^2 + k\) has vertex \((h, k)\)

The Remainder Theorem
If the polynomial \(f(x)\) is divided by \(x - c\), then the remainder is \(f(c)\).

The Factor Theorem
Let \(f(x)\) be a polynomial. If \(f(c) = 0\) then \(x - c\) is a factor of \(f(x)\).
If \(x - c\) is a factor of \(f(x)\) then \(f(c) = 0\).
Asymptotes of Rational Functions

Finding the Horizontal Asymptotes
Consider the rational function,

\[
f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_2 x^2 + b_1 x + b_0}, \quad a_n \neq 0, \quad b_m \neq 0
\]

1. If \( n < m \), the line \( y = 0 \) is the horizontal asymptote of the graph of \( f \).
2. If \( n = m \), the line \( y = \frac{a_n}{b_m} \) is the horizontal asymptote of the graph of \( f \).
3. If \( n > m \), the graph of \( f \) has no horizontal asymptote.

Finding Oblique (Slant) Asymptotes
Let \( N(x) \) and \( D(x) \) be polynomials and consider the rational function \( f(x) = \frac{N(x)}{Q(x)} \).

If the degree of \( N(x) \) is one more than the degree of \( D(x) \), then graph of \( f(x) \) has an oblique (slant) asymptote. To find the equation for the oblique (slant) asymptote first perform long division to obtain

\[
f(x) = \frac{N(x)}{D(x)} = mx + b + \frac{\text{remainder}}{D(x)}
\]

The oblique (slant) asymptote has equation \( y = mx + b \).

Compound Interest
After \( t \) years, the balance, \( A \), in an account with principal \( P \) and annual interest rate, \( r \), is given by

1. \( A(t) = P \left(1 + \frac{r}{n}\right)^{nt} \), for \( n \) compounding periods per year.
2. \( A(t) = Pe^{rt} \), for continuously compounding interest.

Change–of–Base Formula for Logarithms
For any positive real numbers \( M \), \( b \), and \( n \), where \( n \neq 1 \), and \( b \neq 1 \),

\[
\log_b M = \frac{\log_n M}{\log_n b}
\]
Trigonometric Identities

Pythagorean Identities
\[
\tan^2(t) + 1 = \sec^2(t) \quad 1 + \cot^2(t) = \csc^2(t)
\]

Sum and Difference formulas for Sine
\[
\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)
\]
\[
\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)
\]

Sum and Difference formulas for Cosine
\[
\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)
\]
\[
\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)
\]

Sum and Difference formulas for Tangent
\[
\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}
\]
\[
\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}
\]

Double Angle Formulas
\[
\sin(2\theta) = 2\sin(\theta)\cos(\theta)
\]
\[
\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)
\]
\[
\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}
\]

Power-Reducing Formulas
\[
\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}
\]
\[
\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}
\]
\[
\tan^2(\theta) = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}
\]
Half-Angle Formulas

\[
\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}
\]

\[
\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}
\]

\[
\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}} = \frac{1 - \cos(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{1 + \cos(\theta)}
\]

Product to Sum Formulas

\[
\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]
\]

\[
\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]
\]

\[
\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]
\]

\[
\cos(\alpha)\sin(\beta) = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]
\]

Sum to Product Formulas

\[
\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)
\]

\[
\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)
\]

\[
\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)
\]

\[
\sin(\alpha) - \sin(\beta) = 2\sin\left(\frac{\alpha - \beta}{2}\right)\cos\left(\frac{\alpha + \beta}{2}\right)
\]
The Unit Circle