Quadratic Functions

The quadratic function, \( f(x) = ax^2 + bx + c \) has vertex \( \left( -\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right) \).

The quadratic function, \( f(x) = a(x - h)^2 + k \) has vertex \((h, k)\).

**The Remainder Theorem**

If the polynomial \( f(x) \) is divided by \( x - c \), then the remainder is \( f(c) \).

**The Factor Theorem**

Let \( f(x) \) be a polynomial. If \( f(c) = 0 \) then \( x - c \) is a factor of \( f(x) \).

If \( x - c \) is a factor of \( f(x) \) then \( f(c) = 0 \).

Asymptotes of Rational Functions

**Finding the Horizontal Asymptotes**

Consider the rational function,

\[
 f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_2 x^2 + b_1 x + b_0}, \quad a_n \neq 0, \ b_m \neq 0
\]

1. If \( n < m \), the line \( y = 0 \) is the horizontal asymptote of the graph of \( f \).
2. If \( n = m \), the line \( y = \frac{a_n}{b_m} \) is the horizontal asymptote of the graph of \( f \).
3. If \( n > m \), the graph of \( f \) has no horizontal asymptote.

**Finding Oblique (Slant) Asymptotes**

Let \( N(x) \) and \( D(x) \) be polynomials and consider the rational function \( f(x) = \frac{N(x)}{Q(x)} \).

If the degree of \( N(x) \) is one more than the degree of \( D(x) \), then graph of \( f(x) \) has an oblique (slant) asymptote. To find the equation for the oblique (slant) asymptote first perform long division to obtain

\[
 f(x) = \frac{N(x)}{D(x)} = mx + b + \frac{\text{remainder}}{D(x)}
\]

The oblique (slant) asymptote has equation \( y = mx + b \).