Chapter 2 Formulas

The Imaginary Unit
\[ i = \sqrt{-1}, \text{ where } i^2 = -1 \]

Quadratic Functions

\[ f(x) = ax^2 + bx + c \text{ where vertex is } \left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) \]

\[ f(x) = a(x - h)^2 + k \text{ where vertex is } (h, k) \]

The Remainder Theorem
If the polynomial \( f(x) \) is divided by \( x - c \), then the remainder is \( f(c) \).

The Factor Theorem
Let \( f(x) \) be a polynomial. If \( f(c) = 0 \) then \( x - c \) is a factor of \( f(x) \). If \( x - c \) is a factor of \( f(x) \) then \( f(c) = 0 \).

How to Find the Horizontal Asymptotes
Let \( f \) be a rational function. The degree of the numerator is \( n \). The degree of the denominator is \( m \).

1. If \( n < m \), the \( x \)-axis, or \( y = 0 \), is the horizontal asymptote of the graph of \( f \).
2. If \( n = m \), the line \( y = \frac{a_n}{b_m} \) is the horizontal asymptote of the graph of \( f \).
3. If \( n > m \), the graph of \( f \) has no horizontal asymptote.

How to Find Slant Asymptotes
For rational functions, if the degree of the numerator is one more than the degree of the denominator then the equation for the slant asymptote can be found by long division.

\[
\frac{N(x)}{D(x)} = mx + b + \frac{\text{remainder}}{D(x)}
\]

The equation of slant asymptote is \( y = mx + b \)
### Mathematical Equivalent Equations for Variation Statements

<table>
<thead>
<tr>
<th>English Statement</th>
<th>Algebraic Equation</th>
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| **Direct Variation** | $y$ varies directly as $x$.  
$y$ is directly proportional to $x$. |
| | $y = kx$ |
| **Direct Variation with Powers** | $y$ varies directly as $x^n$.  
$y$ is directly proportional to $x^n$. |
| | $y = kx^n$ |
| **Inverse Variation** | $y$ varies inversely as $x$.  
$y$ is inversely proportional to $x$. |
| | $y = \frac{k}{x}$ |
| **Inverse Variation with Powers** | $y$ varies inversely as $x^n$.  
$y$ is inversely proportional to $x^n$. |
| | $y = \frac{k}{x^n}$ |
| **Combined Variation** | $y$ varies directly as $x$ and is  
inversely proportional to $z$. |
| | $y = \frac{kx}{z}$ |
| **Joint Variation** | $y$ varies jointly as $x$ and $z$. |
| | $y = kxz$ |