Mathematics in Sports

Although not always realized, mathematics plays a very important role in sports. Whether discussing a players statistics, a coaches formula for drafting certain players, or even a judges score for a particular athlete, mathematics are involved. Even concepts such as the likelihood of a particular athlete or team winning, a mere case of probability, and maintain equipment are mathematical in nature.

Let's begin by looking at the throwing of a basketball. Now, we can use the equation

\[ f(x) = \left(\frac{-16}{v_o^2 \cos^2 \alpha}\right)x^2 + (\tan \alpha)x + h_o \]

to help figure out the velocity at which a basketball player must throw the ball in order for it to land perfectly in the basket. When shooting a basketball you want the ball to hit the basket at as close to a right angle as possible. For this reason, most players attempt to shoot the ball at a 45° angle. To find the velocity at which a player would need to throw the ball in order to make the basket we would want to find the range of the ball when it is thrown at a 45° angle. The formula for the range of the ball is

\[ Range = \frac{v_o^2 \sin(2\alpha)}{32} \]

But since the angle at which the ball is thrown is 45°, we have

\[ Range = \frac{v_o^2 \sin(2\alpha)}{32} = \frac{v_o^2 \sin(2 \cdot 45)}{32} = \frac{v_o^2}{32} \]

Now, if a player is shooting a 3 point shot, then he is approximately 25 feet from the basket. If we look at the graph of the range function we can get an idea of how hard the player must throw the ball in order to make a 3 point shot.
So, by solving the formula knowing that the range of the shot must be 25 feet we have

\[ 25 = \frac{v_o^2}{32} \]

\[ v_o^2 = 800 \]

\[ v_o \approx 28.2843 \]

So in order to make the 3 point shot, the player must throw the ball at approximately 28 feet per second, 19 mph.

Now let's look at the throwing and hitting of a baseball. The pitcher wants to throw the ball so that he will strike out the batter. If his throw is too high or low then it is a ball and the better still has at least three more opportunities to hit the ball. Similarly, when the batter hits the ball, he wants to hit the ball so that it will be as far away from any of the other players as possible if not outside of the ball field itself. The players must take into consideration the speed and height of the ball to ensure that they will throw or hit it properly. Here is the equation for finding the projectile motion of a baseball will travel:

\[ f(x) = \left( \frac{-16}{v_o^2 \cos^2 \alpha} \right) x^2 + (\tan \alpha) x + h_o \]

where all distances are measured in feet, \( h_o \) is the height from which the ball is thrown, \( \alpha \) is the angle at which the ball is thrown, \( v_o \) is the speed at which the ball is thrown, and \( x \) is the distance that the ball travels. We can find the distance that the ball will travel by saying

\[ y = \frac{v_o^2 \sin(2\alpha)}{32} \]
Now, a batter would be more concerned with the range of the ball, wanting it to travel far enough to allow him to at least make it to first base safely. Let's look at several graphs of the range with different $\alpha$'s and a fixed $v_o$ and $h_o$.

![Graph of range with different $\alpha$'s](image)

The black graph is when $\alpha = 30^\circ$, the blue graph when $\alpha = 45^\circ$, and the red graph when $\alpha = 60^\circ$. So we can see from the graph that an angle of 45° will send the ball the furthest. So, a batter would want to hit the ball as close to a 45° angle as possible, while a pitcher, who is more concerned about the ball veering off path, would want to throw the ball so the ball so that it would travel as close to a straight line as possible.

Now, let's say it is approximately 420 feet from home plate to the edge of a baseball field. The batter wants to hit the ball hard enough so that it will travel out of the field, over the approximately 7 foot wall at the back of the outfield. If the batter hits the ball at a 40° angle and the ball is approximately 5 feet in the air when struck, how hard must he hit the ball in order to have a home run? Remember, that in the projection equation, $f(x)$ is the height of the ball, so now we have

\[
7 = \left(\frac{-16}{v_o^2 \cos^2(40)}\right) 420^2 + (\tan(40)) \cdot 420 + 5
\]

\[
\frac{-16 \cdot 420^2}{(7 - [\tan(40)] \cdot 420 - 5) \cos^2(40)} = v_o^2
\]

\[
v_o^2 \approx 14128.4074
\]

\[
v_o \approx 118.863 \text{ ft/sec}
\]

Therefore we have that the batter must hit the ball at approximately 118 feet per second, which is approximately 81 mph, in order to hit a home run when he hits the ball at an angle of 40°.

We could also look at a sport such as bowling which many people consider to be quite simplistic. However, you must consider the angle of the ball and the velocity with which the ball is thrown when trying to get a strike. The path of a bowling ball, thrown in a straight line, can be represented by the following equation:
\[ f(t) = \left( \frac{v_0}{r} \right) \left( 1 - e^{-rt} \right) \]

where \( v_0 \) is the velocity of the ball, \( t \) is the time in seconds that the ball travels, \( r \) is a constant represents the friction, and \( g(t) \) is the distance in feet that the ball travels after \( t \) seconds.

Now, the length of a blowing lane is approximately 60 feet. Let's say that the friction caused by the bowling ball on the slick surface of the bowling lane is approximately 0.3 and the ball is rolled at approximately 15 mph, or 22 feet per second. Now if we graph this equation we have

![Graph of the equation](image)

So we can see that the bowling ball, if thrown at 15 mph, should make it all the way down the bowling lane.

Mathematics is also used in ranking players and determining playoff scenarios. From something as simple as using a matrix to the formulas used to determine a players or teams statistics, mathematics is an integral part of this system. For example, in the olympics, most sports have players draw numbers to see who they will be competing against. If there are \( 2^k \) contestants then all athletes participate in the first round of play, if not, then some of the participants enter during the second round of play. The number of athletes entering during the second round of play will be \( 2^k - n \), where \( n \) is the number of contestants. Rankings are also an important aspect of sports. In sports such as tennis, when rating athletes, an integral estimator is used which is based on a players performance in a series of matches over a certain period of time. Even horse racing uses mathematics to rank the horses based on how well the have performed in previous matches, and these rankings go into determining the value of a horse when a bet is placed. Mathematics is very prevalent in sports, from the most complex of formulas to the simplest ideas such as betting.